Unsteady Flow along a Stretching Sheet in the Presence of Transverse Magnetic Field

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Abstract—*This paper studied the recent development of The effect of thermal radiation using the nonlinear Rosseland approximation, The paper briefly explains the relevant knowledge about relationship between film thickness E and the unsteadiness parameter S is found; the effects of unsteadiness parameter . Fluid flow with heat and mass transfer towards a stagnation point on a vertical plate. In this study, we consider both strong concentrations (n = 0) and weak concentrations (n = 1/2).). Flow agree well with the exact solutions for traditional Navier-Stokes equation with no slip boundary conditions. The governing partial differential equations are transformed into ordinary differential equations by using similarity transformations. Results of the flow characteristics are in good agreement with the experimental results given in the literature. The results for small Prandtl number (Pr).The resulting similarity equations are solved numerically using Runge-Kutta-Fehlberg method. A parametric study illustrating the influence of the unsteadiness parameter and Prandtl number on the fluid velocity as well as temperature is conducted.*

Keywords: Unsteady stretching surface, similarity transformation; heat transfer, review, Prandtl number.

1. I-INTRODUCTION

Fluid flow and heat transfer in a thin liquid film over a stretching sheet find many applications in industrial processes for this purpose, different methods have been developed and can be grouped in four main categories: analytical ,semi analytical, numerical and Hybrid Methods –combinations of three .Numerical Methods are very flexible to various geometries, include nonlinear and non-Homogeneous material. In the pioneering work of Wang [1], the flow in a thin liquid film past an unsteady stretching sheet was investigated. Recently, several authors extended Wang's work by including the non-Newtonian effect of fluid, heat transfer and the thermo capillarity effect [2-4]. However, Numerical solutions provided are the most in the above studies. The objective of the present study is to investigate the problem of heat transfer over a stretching sheet with Newtonian heating (NH) and to see the effect of various values of Prandtl number

However the Newtonian heating problem has been examined by Lesnic et al. [5] to study the free convection boundary layer along a vertical surface *embedded in a porous medium. Salleh et al. [5,7] investigated the forced convection boundary flow at a forward stagnation point with Newtonian heating as well as the boundary layer flow and heat transfer over a stretching sheet with Newtonian heating, respectively. Recently, Hayat et al. [8] addressed the effect of Newtonian heating on the boundary layer flow and heat transfer in the second grade fluid*

2. MATHEMATICAL FORMULATION

A. Governing Equations and Boundary Conditions

Let us consider a thin elastic sheet emerging from a narrow slit at the origin of a Cartesian co-ordinate system. The continuous sheet at y=0 is parallel with the x-axis and moves in its own plane with the velocity

$$U(x,t)=bx/(1-\alpha t) \tag{1}$$

where b and D are both positive constants with dimension per time. The surface temperature s T of the stretching sheet is assumed to vary with the distance x from the slit as

$$T_s(x,t) \backslash=T_0-T_{ref}[bx^2/2v](1-\alpha t)^{3/2} \tag{2}$$

where T_0 is the temperature at the slit and T_{ref} can be taken as a constant reference temperature such that $0 \le T_{ref} \le T_0$. The term $bx^2/v(1-\alpha t)$can be recognized as the Local Reynolds number based on the surface velocity U Equation (1) for the velocity of the sheet U(x,t) reflects that the elastic sheet which is fixed at the origin is stretched by applying a force in the positive x-direction and the effective stretching rate b/(1-αt)increases with time as $0\le\alpha\le1$. With the same analogy the expression for the surface temperature $T_s(x,t)$ Txt given by (2) represents a situation in which the sheet temperature decreases from T_0 at the slit in proportion to x^2 and such that the amount of temperature reduction along the sheet increases with time. The applied transverse magnetic field is assumed to be of variable kind and is chosen in its special form as

$$B(x,t)=B_0(1-\alpha t)^{1/2} \tag{3}$$

The particular form of the expressions for U(x ,t) Ts(x ,t) and B(x ,t) are chosen so as to facilitate the construction of a new similarity transformation, which enables transforming the governing partial differential equations of momentum and heat transport into a set of nonlinear ordinary differential equations.

Consider a thin elastic liquid film of uniform thickness h (t) lying on the horizontal stretching sheet (Fig. 1). The x- axis is chosen in the direction along which the sheet is set to motion and the y-axis is taken perpendicular to it. The fluid motion within the film is primarily caused solely by stretching of the sheet. The sheet is stretched by the action of two equal and opposite forces along the x-axis. The sheet is assumed to have velocity U as defined in (1) and the flow field is exposed to the influence of an external transverse magnetic field of strength B as defined in (3). We have neglected the effect of latent heat due to evaporation by assuming the liquid to be nonvolatile.

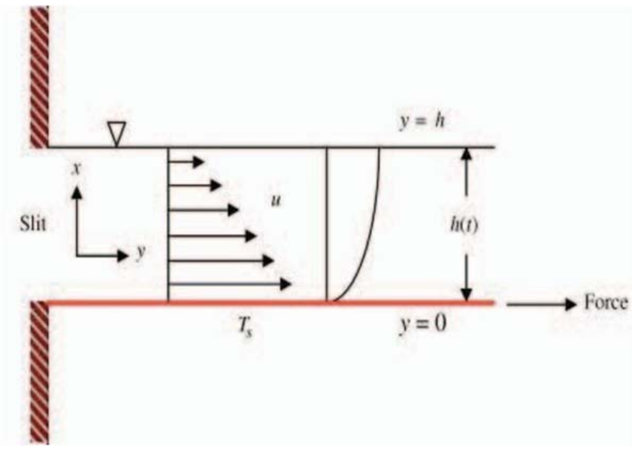

Fig. 1: Physical Modal

Further, the buoyancy is neglected due to the relatively thin liquid film, but it is not so thin that intermolecular forces come into play. The velocity and temperature fields of the liquid film obey the following boundary layer equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}=0, \tag{4}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2} \tag{5}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{6}$$

Following Roseland approximation for radiation (see [10]) the radiative heat flux r q and is modeled as

$$q_r=-\frac{4\sigma}{3k}\frac{\partial T^4}{\partial y} \tag{7}$$

where σ are the Stefan-Boltzmann constant and k is the mean absorption coefficient. Assuming that temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of temperature $T^4 \equiv 4\, T_\infty^3 T-3\, T^4_\infty$

$$\frac{16\sigma T\infty 3}{3k}\frac{\partial^2 t}{\partial y^2} \tag{8}$$

where T is the temperature across the boundary layer. We have supposed T as x-dependent, and in view of (8), (6) reduces to

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\left(\frac{k}{\rho C_p}+\frac{16\sigma T\infty 3}{3k\rho C_p}\right)\frac{\partial^2 t}{\partial y^2} \tag{9}$$

The pressure in the surrounding gas phase is assumed to be uniform and the gravity force gives rise to a hydrostatic pressure variation in the liquid film. In order to justify the boundary layer approximation, the length scale in the primary flow direction must be significantly larger than the length scale in the cross-stream direction. We choose the representative measure of the film thickness to be $(v/b)^{1/2}$ so that the scale ratio is large enough i.e. $x/(v/b)^{1/2} > 1$ This choice of length scale enables us to employ the boundary layer approximations. Further it is assumed that the induced magnetic field is negligibly small. The associated boundary conditions are given by

$$u=U,\quad v=T=Ts \qquad \text{at}\quad y=0 \tag{10}$$

$$\frac{\partial u}{\partial y}=\frac{\partial T}{\partial y}=0 \qquad \text{at}\qquad y=h \tag{11}$$

$$v=\frac{dh}{dt} \qquad \text{at}\qquad y=h \tag{12}$$

We make a note that the mathematical problem is implicitly formulated only for x≥0 . Further it is assumed that the surface of the planar liquid film is smooth so as to avoid the complications due to surface waves. The influence of interfacial shear due to the quiescent atmosphere, in other words the effect of surface tension is assumed to be negligible. The viscous shear stress $\tau=\mu\left(\frac{\partial u}{\partial y}\right)$ and that the alphabetic free surface at (y=h) ,the heat flux $q=-k\left(\frac{\partial T}{\partial y}\right)$ vanishes.

$$S\left(f'+\frac{\eta}{2}f''\right)+(f')^2-ff''=f'''-Mf', \tag{13}$$

$$\Pr\left[\frac{s}{2}(3\theta+\eta\theta')+2f'\theta-\theta' f\right]=\theta''(1+Nr), \tag{14}$$

$$f'(0)=1, f(0)=0, \theta(0)=1 \text{ and } f''(\beta)=0, \theta'(\beta)=0 \tag{15}$$

$$f(\beta)=s\beta/2 \tag{16}$$

3. NUMERICAL APPROACH

The non-linear differential equations (13) and (14) with subjected to the boundary conditions (15)-(16) were solved numerically using by efficient Runge-Kutta-Fehlberg method (Conte et al. [9]). In this method third-order non-linear equation(13), second order equation (14) have been reduced to five ordinary differential equations as follows:

$$\frac{df_0}{d\eta}=f_1, \tag{17}$$

$$\frac{df_1}{d\eta}=f_2, \tag{18}$$

$$\frac{df_2}{d\eta}=S\left(f_1+\frac{\eta}{2}f_2\right)+(f_1)^2-f_0f_2+Mf_1, \tag{19}$$

$$\frac{d\theta_0}{d\eta}=\theta_1, \qquad (20)$$

$$\frac{d\theta_1}{d\eta}=\frac{1}{1+Nr}\Pr\left[\frac{S}{2}(3\theta_0+\eta\theta_1)+(2f_1\theta_0\theta_1 f_0\right] \qquad (21)$$

Corresponding boundary conditions take the form,

$f_1(0)$=1,$f_0(0)$ =0, $\theta_0(0)$ =1, and

$f_2(\beta)$ =0, $\theta_1(\beta)$ =0, (22)

$f_0(\beta)$=Sβ/2 (23)

where$f_0(\eta)$ = f(η) and $\theta_0(\eta)=\theta(\eta)$ The above boundary value problem is first converted into an initial value problem by appropriately guessing the missing slopes

$f_2(0)$ and $\theta_1(0)$. The resulting IVP is solved by the shooting method for a set of parameters appearing in the governing equations and a known value of f_2 (0) andθ_1 (0).The value of β is so adjusted that condition (23) holds. This is done on the trial and error basis. The value for which condition (23) holds is taken as the appropriate film thickness and the IVP is finally solved using this value of β. The step length of h = 0.01 is employed for the computation purpose. The convergence criterion largely depends on fairly good guesses of the initial conditions in the shooting technique. The iterative process is terminated until the relative difference between the current and the previous iterative values of f(β)matches with the value of Sβ/2 up to a tolerance of 10^{-6} Once the convergence is achieved we integrate the resultant ordinary differential equations using standard fourth order Runge-Kutta method with the given set of parameters to obtain the required solution.

4. RESULTS AND DISSCUSSION

The problem of laminar flow of liquid film due to an unsteady stretching sheet has been analyzed. A suitable similarity transformation is adopted to transform the non-linear ordinary differential equations. The resultant boundary value problem is solved by the efficient shooting method. It might be worthy to mention that the solution exists only for small value of unsteadiness parameter 0 ≤S≤2 Moreover, when S →0 the solution approaches the analytical solution obtained by Crane [10] with infinitely thick layer of fluid (β→∞). E of The other limiting solution corresponding to S →2 represents a liquid film of infinitesimal thickness (β→0). The numerical results are obtained for 0 ≤S≤2 . Present results compared with some of the earlier published results Anderson, Bilchenko et al. [11,12]. The effects of various parameters influencing the dynamics are shown in Figs. 2-5. Figures 2 and 3 depict the effect of M on temperature profiles for two different values of S. The results show that the thermal boundary layer thickness increases with the increasing values of M. The increasing frictional drag due to the Lorentz force is responsible for increasing the thermal boundary layer thickness.

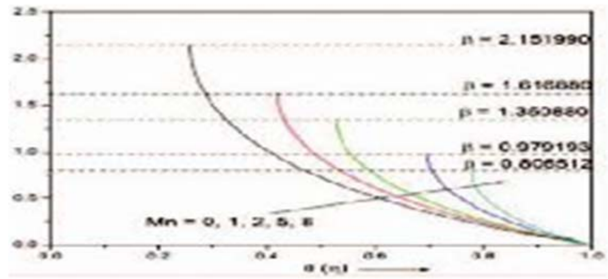

Fig. 2

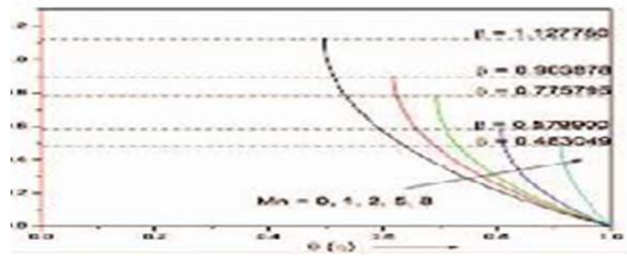

Fig. 3

Figures 4 and 5 demonstrate the effect of Prandtl number Pr on the temperature profiles for two different values of unsteadiness parameter S. These plots reveal the fact that for a particular value of Pr the temperature increases monotonically from the free surface temperature Ts to wall velocity T_0 as observed in Anderson et al [13]. The thermal boundary layer thickness decreases drastically for high values of Pr i.e., low thermal diffusivity.

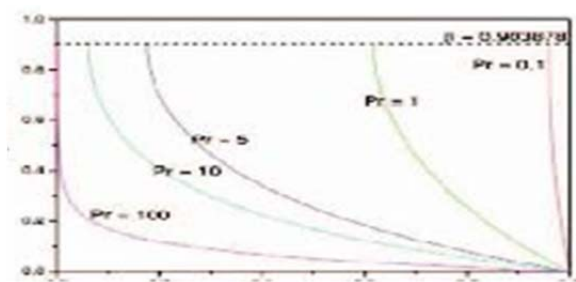

Fig.4

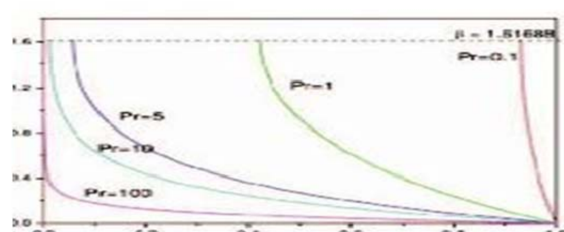

Fig. 5

The velocity and temperature profiles for a thin film over an unsteady stretching sheet are presented in Fig.5 by Sidorov [14].

5. CONCLUSION

In this paper, we investigate the combined effects of variable transverse magnetic field and thermal radiation effect. Present results reveal that the magnetic field effects play significant role on controlling the heat transfer from stretching sheet to the liquid film. The important findings of the present analysis are:

The effect of transvers magnetic field on a viscous incompressible electrically conducting fluid is to suppress the velocity field, which in turn causes the enhancement of the temperature field.

An increasing Prandtl number Pr causes dimension in the thickness of thermal boundary layer.

The effect of the thermal radiation parameter produces a significant increase thickness of the thermal boundary layer of the liquid film and so temperature increase

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